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Three-Phase Voltage-Source Converters

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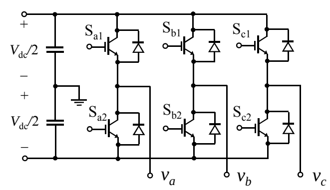
Outline

- Basic Operation & Applications
- Pulse-Width Modulation
- AC-Side Current Control
- DC-Link Voltage Regulation

Three-Phase VSC Circuit

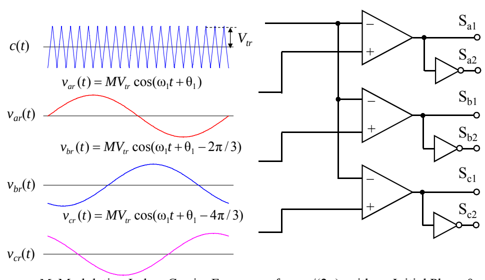


- A Versatile Interface between DC and Three-Phase AC
- Middle of DC Link is Usually Used as Reference Point
- Inductive Elements at AC Terminals Required
- Bidirectional Power Flow Capabilities

Applications in Power Systems

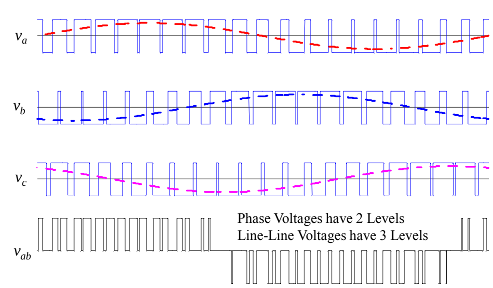
- Renewable Energy Integration into the Grid
  - Including Energy Storage
- High-Voltage DC Transmission
- Reactive Power Compensation
  - Including Harmonics as Active Power Filter
- Dynamic Voltage Restorer (DVR)
- Unified Power Flow Controller (UPFC)
- Other FACTS Devices

Carrier-Based PWM

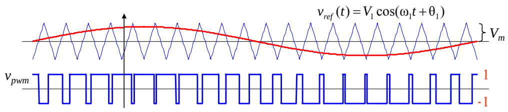


M: Modulation Index; Carrier Frequency $f_c = \omega_c/(2\pi)$, with an Initial Phase θ_c.

Phase and Line-Line Voltages



PWM Spectrum



$$v_{pwm}(t) = M\cos(\omega_1 t + \theta_1) + \sum_{m=1}^{\infty}\frac{4}{m\pi}\cdot J_0\left(\frac{m\pi M}{2}\right)\cdot\sin\frac{m\pi}{2}\cdot\cos[m(\omega_c t + \theta_c)]$$
$$+\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{4}{m\pi}\cdot J_n\left(\frac{m\pi M}{2}\right)\cdot\sin\frac{(m+n)\pi}{2}\cdot\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

$M = V_1/V_m$: Modulation Index
$\omega_1 = 2\pi f_1$: Reference Frequency
$\omega_c = 2\pi f_c$: Carrier Frequency
θ_1: Initial Phase Angle of Reference
θ_c: Initial Phase Angle of Carrier
$J_n(x)$: Bessel Function of the 1st Kind

Spectral Composition

Fundamental (f_1) Carrier (f_c) and Its Harmonics ($2f_c$, $3f_c$, ……)

$$v_{pwm}(t) = M\cos(\omega_1 t + \theta_1) + \sum_{m=1}^{\infty}\frac{4}{m\pi}\cdot J_0\left(\frac{m\pi M}{2}\right)\cdot\sin\frac{m\pi}{2}\cdot\cos[m(\omega_c t + \theta_c)]$$
$$+\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{4}{m\pi}\cdot J_n\left(\frac{m\pi M}{2}\right)\cdot\sin\frac{(m+n)\pi}{2}\cdot\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

Sideband Components: $mf_c \pm f_1$, $mf_c \pm 2f_1$ $mf_c \pm 3f_1$ ……

Alternative Form:

$$v_{pwm}(t) = M\cos(\omega_1 t + \theta_1) +$$
$$\sum_{m=1}^{\infty}\sum_{n=0,\pm1,\ldots}^{\pm\infty}\frac{4}{m\pi}\cdot J_n\left(\frac{m\pi M}{2}\right)\cdot\sin\frac{(m+n)\pi}{2}\cdot\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

Phase Voltage Spectra

$$v_a(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$
$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

$$v_b(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1 - \frac{2\pi}{3}) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$
$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1 - \frac{2\pi}{3})]$$

$$v_c(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1 - \frac{4\pi}{3}) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$
$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1 - \frac{4\pi}{3})]$$

Carrier and its harmonics are the same in all three phases; Sideband components are phase shifted by multiple times of that between the fundamental components.

Line-Line Voltage Spectra

$$v_a(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$
$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

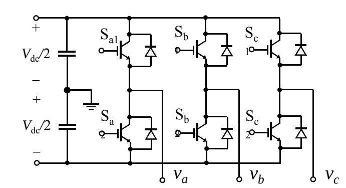
$$v_b(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1 - \frac{2\pi}{3}) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$
$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1 - \frac{2\pi}{3})]$$

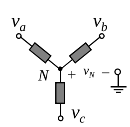
$$v_{ab}(t) = \frac{\sqrt{3}V_{dc}}{2}M\cos(\omega_1 t + \theta_1 + \frac{\pi}{6})$$ ← Fundamental

$$+\frac{2\sqrt{3}V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=\pm1,\pm2,\ldots\\ n\neq\pm3k}}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1) \pm \frac{\pi}{6}]$$

Sideband Components

AC Neutral Voltage





$$v_N = \frac{v_a + v_b + v_c}{3}$$

Assume Balanced AC with a Floating Neutral Point, N:

$$I_a(s) + I_b(s) + I_c(s) = 0 \Leftarrow i_a(t) + i_b(t) + i_c(t) = 0 \qquad v_N(t) = \frac{v_a(t) + v_b(t) + v_c(t)}{3}$$

$$\frac{V_a(s) - V_N(s)}{Z(s)} + \frac{V_b(s) - V_N(s)}{Z(s)} + \frac{V_c(s) - V_N(s)}{Z(s)} = 0 \Rightarrow V_N(s) = \frac{V_a(s) + V_b(s) + V_c(s)}{3}$$

Common-Mode Voltage

$$v_{cm} = \frac{v_a + v_b + v_c}{3}$$

- Defined as the Common Component of Output Voltages
  - The Same as the Load Neutral Voltage
  - Also called Zero-Sequence Voltage

$$v_a(t) = \frac{V_{dc}}{2}M\cos(\omega_1 t + \theta_1) + \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$ ← Carrier and Its Harmonics

$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm1}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

Sideband Components Centered around the Carrier Frequency and its Harmonics

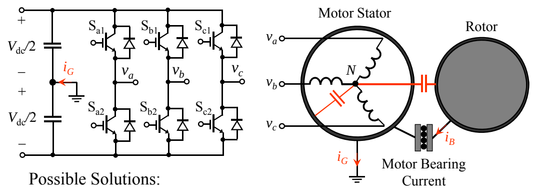
$$v_{ab}(t) = \frac{\sqrt{3}V_{dc}}{2}M\cos(\omega_1 t + \theta_1 + \frac{\pi}{6})$$
$$+\frac{2\sqrt{3}V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=\pm1,\pm2,\ldots\\ n\neq\pm3k}}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1) \pm \frac{\pi}{6}]$$

$$v_{cm}(t) = \frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(\frac{m\pi}{2}M\right)\sin\frac{m\pi}{2}\cos[m(\omega_c t + \theta_c)]$$ ← Carrier and Its Harmonics

$$+\frac{2V_{dc}}{\pi}\sum_{m=1}^{\infty}\sum_{n=\pm3,\pm6,\pm9,\ldots}^{\pm\infty}\frac{1}{m}J_n\left(\frac{m\pi}{2}M\right)\sin\frac{(m+n)\pi}{2}\cos[m(\omega_c t + \theta_c) + n(\omega_1 t + \theta_1)]$$

Effects of CM Voltage

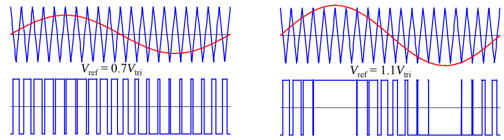
- Common-Mode (Ground) Current – EMI Problem
- Motor Bearing Current – Motor Reliability

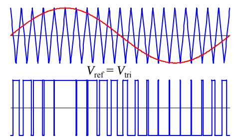


Possible Solutions:

- Common-Mode Voltage Filtering
- CM Voltage Reduction by Circuit Topology and PWM Techniques
- Ceramic Bearing (High Performance Systems e.g. Aerospace)

Maximum Output Voltage



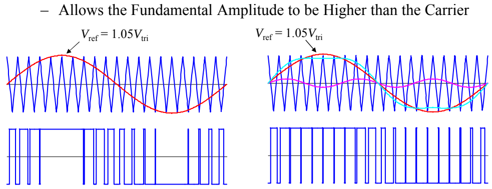


- Maximum Output Voltage
  - Achieved when $M = 1$

    $V_{a,\max} = V_{dc}/2$
- Overmodulation
  - Nonlinear Gain – Saturation
  - Increased Harmonics

3rd Harmonic Injection

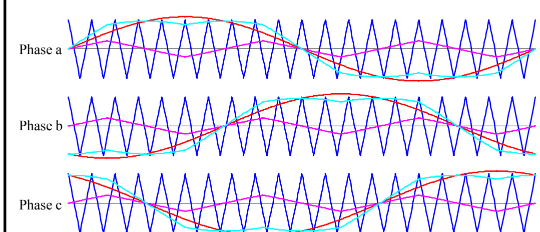
- 3rd Harmonics (of the Fundamental) are of Zero-Sequence
  - Identical for All Three Phases: $3\times120^\circ = 360^\circ$
- Adding the Same 3rd-Order Harmonic to the References
  - Doesn't Affect Load Phase Voltages
  - Allows the Fundamental Amplitude to be Higher than the Carrier



Overmodulation Linear Operation

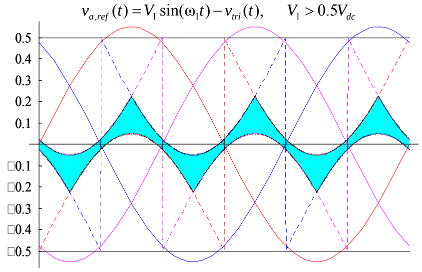
Triplen Harmonics Injection

All triplen harmonics are zero-sequence components, hence can be injected without affecting the load phase voltage. A special case is a triangular wave.



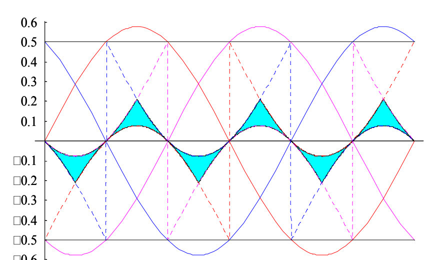
Limits for Harmonics Injection

$$v_{a,ref}(t) = V_1 \sin(\omega_1 t) - v_{tri}(t), \quad V_1 > 0.5V_{dc}$$



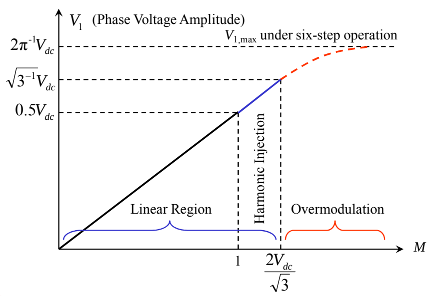
$v_{tri}(t)$ must fall in the shadowed area in order to avoid distortion

Maximal Output Voltage

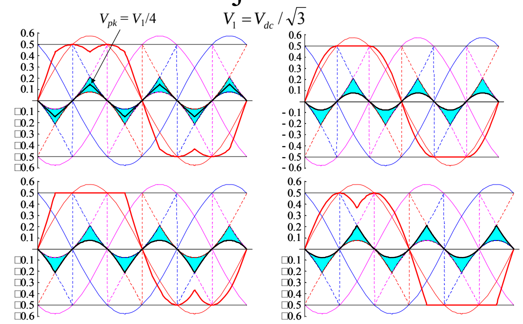


$$v_{ref}\left(\frac{iT_1}{6}\right) = V_1 \sin\frac{\pi}{3} = \frac{\sqrt{3}V_1}{2} \Rightarrow V_{1,\max} = \frac{V_{dc}}{\sqrt{3}}$$

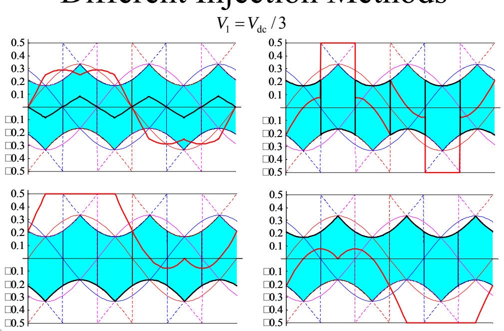
Maximal Output Voltage



Different Injection Methods



Different Injection Methods

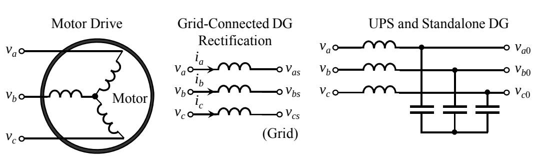


Effects on Phase Voltages

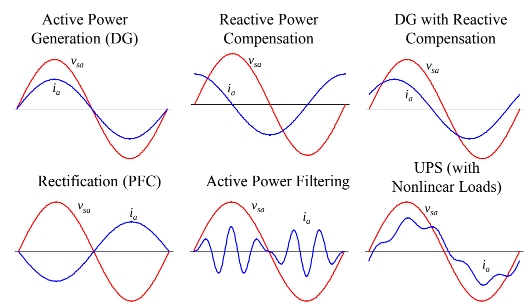
- Phase Voltages Contain Additional Harmonics under Harmonic Injection
  - Injected Harmonics and Sideband Components
  - Identical in All Three Phases
    - Hence Cancel in Line-Line Voltages
- These Harmonics Become Part of Common-Mode Voltage
  - Relatively Low Frequency and Magnitude Compared to the CM Voltage Generated by PWM
- Phase Voltage Spectra can also be Obtained by Double Fourier Series Analysis
  - Closed-Form Results have been Reported in the Literature
- Optimal Harmonic Injection

AC Current Control

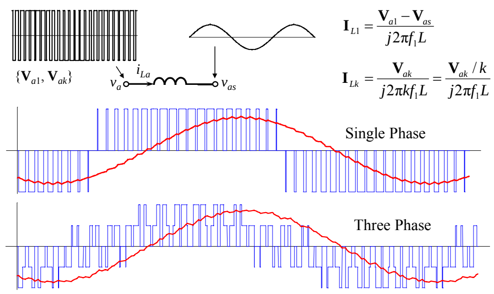
- DC-AC Converter (Inverter) Current Control
  - Torque Control in Motor Drives
  - Active and Reactive Power Control in Grid-Connected DG
  - Current-Mode Control in UPS and Standalone DG
- AC-DC Converter (Rectifier) Current Control
  - Unity Input Power Factor
  - Regulation of DC Voltage



Example Current Waveforms



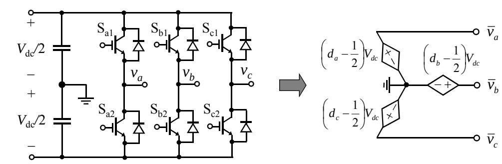
Current Control Principles



$$I_{L1} = \frac{V_{a1} - V_{as}}{j2\pi f_1 L}$$

$$I_{Lk} = \frac{V_{ak}}{j2\pi k f_1 L} = \frac{V_{ak}/k}{j2\pi f_1 L}$$

Averaged Modeling

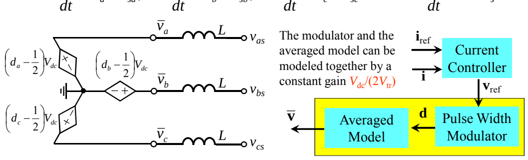


- d_a, d_b, and d_c is the Duty Ratio of the Upper Switch in Phase a, b, and c, Respectively
- Averaging Removes Switching Ripple
  - Resulting Model is in General Valid at Frequencies Lower than Half the Switching (Carrier) Frequency

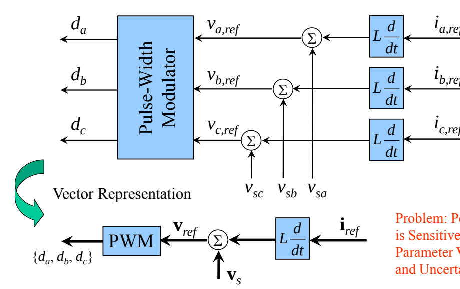
Basic Control Structure

- Objective: Control Phase Currents i_a, i_b, and i_c to Follow Given References $\{i_{a,\,ref}, i_{b,\,ref}, i_{c,\,ref}\}$
- Assumption: Terminal Voltages v_{sa}, v_{sb}, and v_{sc} are Known
- Current Responses Governed by

$$L\frac{di_a}{dt} = v_a - v_{sa},\quad L\frac{di_b}{dt} = v_b - v_{sb},\quad L\frac{di_c}{dt} = v_c - v_{sc} \quad \Rightarrow \quad L\frac{d\mathbf{i}}{dt} = \mathbf{v} - \mathbf{v}_s$$

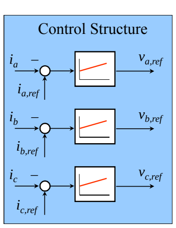


Feedforward Control



Problem: Performance is Sensitive to Parameter Variations and Uncertainty

Linear Feedback Control



PI Regulator:

$$v_{ref} = K_p \Delta i + K_i \int_0^t \Delta i dt$$

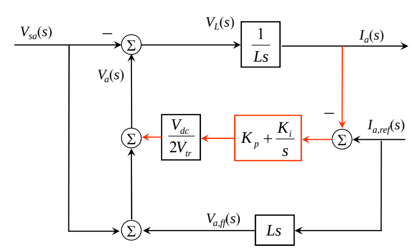
- One Linear (PI) Regulator per Phase
- Simple, Robust
- Limited Performance
  - Existence of Control Error

$$V(s) = [I_{ref}(s) - I(s)] \cdot \left(K_p + \frac{K_i}{s}\right) \cdot \frac{V_{dc}}{2V_{tr}}$$

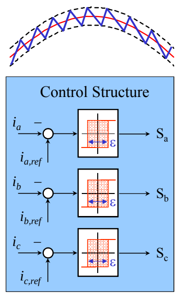
$$I(s) = \frac{V(s) - V_s(s)}{Ls}$$

$$I(s) = \frac{I_{ref}(s)\left(K_p + \frac{K_i}{s}\right)\frac{V_{dc}}{2V_{tr}} - V_s(s)}{Ls + \left(K_p + \frac{K_i}{s}\right)\frac{V_{dc}}{2V_{tr}}}$$

Feedback + Feedforward



Nonlinear (Hysteretic) Control

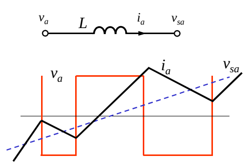


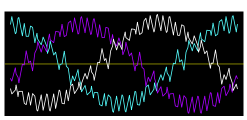
- Features
  - Direct Generation of Switch Gate Signals
    - No Need for PWM
  - Simple, Robust
  - Stable Operation, Fast Response
  - Variable Switching Frequencies
  - Not Suitable for Digital Control
- Possible Improvement
  - Variable Hysteretic Bands to Reduce Switching Frequency Variation
  - Space Vector-Based Hysteretic Control

Practical Control Methods

- Control in DQ Reference Frame
  - Balanced Three-Phase References Become DC in DQ Frame
  - No Steady-State Tracking Error
- Resonant Feedback Compensation
  - Compensator has a Resonance at the Line Frequency
  - Improve Tracking of Inductor Current at the Line Frequency

Ripple Current and *L* Selection

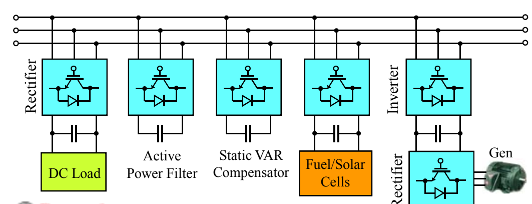




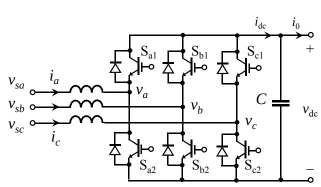
- Inductor Currents Contain Ripple Components at the Switching Frequency
- Amplitude $\propto (Lf_s)^{-1}$
  - Varies over a Line Cycle
- Effects of Current Ripple
  - Contribute to System Harmonics
  - Increase Converter Losses
- Inductor Design Considerations
  - Performance vs. Cost & Size

DC-Link Voltage Control

- Applications Requiring Control of the DC-Link Voltage
  - Power Factor Corrected Rectifiers
  - Active Power Filters
  - Static VAR Compensators
  - Grid Interface for Fuel Cells, Solar, and Wind Power



DC-Link Voltage Model



$$C\frac{dv_{dc}}{dt} = i_{dc} - i_0$$

$$\bar{i}_{dc} = d_a i_a + d_b i_b + d_c i_c$$

$$\bar{v}_a = \frac{d_a v_{dc}}{2} - \frac{(1-d_a)v_{dc}}{2}$$

$$d_a = \frac{\bar{v}_a}{v_{dc}} + \frac{1}{2}$$

$$\bar{i}_{dc} = \frac{\bar{v}_a i_a + \bar{v}_b i_b + \bar{v}_c i_c}{v_{dc}} + \frac{\overbrace{i_a + i_b + i_c}^{0}}{2} = \frac{\bar{v}_a i_a + \bar{v}_b i_b + \bar{v}_c i_c}{v_{dc}} \approx \frac{\bar{v}_{sa} i_a + \bar{v}_{sb} i_b + \bar{v}_{sc} i_c}{v_{dc}}$$

Power Balance: $\bar{i}_{dc} v_{dc} = \bar{v}_a i_a + \bar{v}_b i_b + \bar{v}_c i_c$

$$C\frac{d\bar{v}_{dc}}{dt} = \frac{\mathbf{v}^T\mathbf{i}}{\bar{v}_{dc}} - i_0$$

V_{dc} Response to Phase Currents

- Assume Balanced Sinusoidal Source Voltages
- Only Active Component of Phase Currents Affects Average of V_{dc}
- Reactive Component doesn't Affect V_{dc}
- Unbalanced Fundamental Components (Active or Reactive) Lead to 2nd Harmonic in V_{dc}
- Balanced $(6k \pm 1)^{\text{th}}$ Harmonic Currents Generates $6k^{\text{th}}$ Harmonic in V_{dc}

$$v_{sa}(t) = V_s\cos(\omega_1 t), \quad v_{sb}(t) = V_s\cos(\omega_1 t - 2\pi/3), \quad v_{sc}(t) = V_s\cos(\omega_1 t - 4\pi/3)$$

$$\cos x\cos x + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x - \frac{4\pi}{3}\right)\cos\left(x - \frac{4\pi}{3}\right) = \frac{3}{2}$$

$$\sin x\cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x - \frac{4\pi}{3}\right)\cos\left(x - \frac{4\pi}{3}\right) = 0$$

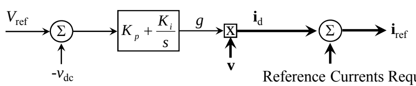
$$\sin 5x\cos x + \sin 5\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin 5\left(x - \frac{4\pi}{3}\right)\cos\left(x - \frac{4\pi}{3}\right) = \frac{3}{2}\cos 6x$$

Voltage Dynamics and Control

Assume Active Component of Phase Current $\mathbf{i}_d = g\mathbf{v}$ ⇒ $C\frac{d\bar{v}_{dc}}{dt} = \frac{\mathbf{v}^T\mathbf{i}}{\bar{v}_{dc}} - i_0 = \frac{3gV_{rms}^2}{\bar{v}_{dc}} - i_0$

V_{rms} = Input Phase RMS Voltage

- g is the Control Variable
  - Represents Voltage Compensator Output
- Dynamics Described by a Nonlinear Model
  - Linearization about a Steady-State Operation Point



Reference Currents Required for System Control Functions

Reading & HW Assignments

- Holmes and Lipo, Pulse Width Modulation for Power Converters, Chapters 5&6
- Consider A Three-Phase Solar Inverter Switched at 20 kHz and with a 350 V DC Input. The Inverter is Connected to a 120 V (Phase RMS) Grid through a 1 mH (per Phase) Inductor, and Supplies 10 kW Active Power to the Grid. Calculate
  - The Modulation Index M
  - The Amplitude of Phase Harmonic Current at 20 kHz, 20 kHz ± 60 Hz, 20 kHz ± 120 Hz, 20 kHz ± 180 Hz; and 40 kHz, 40 kHz ± 60 Hz, 40 kHz ± 120 Hz, 40 kHz ± 180 Hz